

# Fermion localization on branes with generalized dynamics

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## Abstract

In this work we reinvestigate braneworld models with nonstandard dynamics. We choose a specific model diffused in the literature and focus our attention on the matter energy density, the energy of system, the Ricci scalar and the thin brane limit. As the model is classically stable and capable of localize gravity, as a natural extension we address the issue of fermion localization of fermions on a thick brane constructed out from one scalar field with nonstandard kinetic terms coupled with gravity. The contribution of the nonstandard kinetic terms in the problem of fermion localization is analyzed. It is found that the simplest Yukawa coupling  $\eta\bar{\Psi}\phi\Psi$  support the localization of fermions on the thick brane. It is shown that the zero mode for left-handed can be localized on the thick brane depending on the values for the coupling constant  $\eta$ . Furthermore, we present an argument that could jeopardize the results of previous work on massive modes.

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## I. INTRODUCTION

In the last decade, the braneworld scenario has attracted a lot of interests for it gives an effective way to solve the hierarchy problem by introducing two 3-branes which are embedded in a five-dimensional anti-de Sitter ( $AdS_5$ ) space-time [1]. As another attractive property, the Newtonian law of gravity with a correction is also given in this braneworld scenario [2]. In the Randall-Sundrum model [1], we can further add scalar fields [3] with usual dynamics and allow them to interact with gravity in the standard way. In this scenario, the smooth character of the solutions generate thick brane with a diversity of structures [4]-[7]. In the braneworld scenarios, an important issue is how gravity and different observable matter fields of the Standard Model of particle physics are localized on the brane. It has been shown that, in the Randall-Sundrum model in 5-dimensional space-time, graviton and spin 0 field can be localized on a brane with positive tension [2],[8]-[9]. Moreover, spin 1/2 and 3/2 can be localized on a negative-tension brane [9]. The localization problem of spin-1/2 fermions on thick branes is interesting and important [8]-[30]. In order to achieve localization of fermions on a brane with positive tension, it seems that additional interactions except the gravitational interaction must be including in the bulk.

On the other hand, the first recent observations [31]-[32] have led us with the intriguing fact that the Universe is presently undertaking accelerated expansion. These information directly contributed to establish some important advances in cosmology, one of them being the presence of dark energy. The presence of dark energy has opened some distinct routes of investigations. In recent years, there appeared some interesting models with noncanonical dynamics with focus on early time inflation or dark energy [33]-[36], as for instance, the so-called k-fields, first introduced in the context of inflation [36] and the k-essence models, suggested to solve the cosmic coincidence problem [35],[37]. The interaction between dark energy and fermion fields has already a precedent in the cosmology context [38]. In the context of braneworld scenario, the effect of general brane kinetic terms for bulks scalars, fermions and gauge bosons in theories with and without supersymmetry has already been analyzed in [39]. We believe that the conditions for obtaining normalizable zero modes in a brane model with generalized dynamics deserves to be more explored.

In this paper, we reinvestigate a braneworld model with nonstandard dynamics. The model  $\mathcal{L} = K(X) - V(\phi)$ , where  $K(X) = X + \alpha|X|X$  (type I model in Ref. [33]) is

considered. We will focus our attention mainly on the matter energy, the energy of system, the Ricci scalar and the thin brane limit. As the model is classically stable and capable of localize gravity, additionally we address ourselves to the issue of fermion localization on a thick brane constructed out from one scalar field with nonstandard kinetic terms coupled to gravity. We use the analytical expressions for small  $\alpha$  and investigate the contribution of this nonstandard kinetic terms in the problem of fermion localization. We find that the simplest Yukawa coupling  $\eta\bar{\Psi}\phi\Psi$ , where  $\eta$  is the coupling constant, which allows left-handed to posses a localized zero mode on the thick brane under some conditions on the value for the coupling constant  $\eta$ . For  $\alpha$ , which is not necessarily small, we can not get any expressions for the solution of this model, for this case the numerical study is used. The numerical results bear out our results for small  $\alpha$ . The organization of this paper is as follows: in Sec. II, we review and finish the analysis of brane model with generalized dynamics developed by Bazeia and collaborators [33]. In Sec. III, we study the localization of spin-1/2 fermion for this model, we analyze the essential conditions for the localization with the simplest Yukawa coupling and present an argument that could jeopardize the results of previous work on massive modes. Finally, our conclusions are presented in Sec. IV.

## II. REVIEW OF SYSTEMS WITH GENERALIZED DYNAMICS

The action for this kind of system is described by [33]

$$S = \int d^5x \sqrt{|g|} \left[ -\frac{1}{4}R + \mathcal{L}(\phi, X) \right], \quad (1)$$

where  $g \equiv \text{Det}(g_{ab})$  and  $X = \frac{1}{2} \nabla^a \phi \nabla_a \phi$ . The line element of the five-dimensional space-time can be written as

$$ds^2 = g_{ab} dx^a dx^b = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (2)$$

where we are using the five-dimensional Newton constatat  $4\pi G^{(5)} = 1$ ,  $y = x^4$  is the extra dimension (the Latin indices run from 0 to 4),  $\eta_{\mu\nu}$  is the Minkowski metric with signature  $(+, -, -, -)$  and  $e^{2A}$  is the so-called warp factor (the Greek indices run from 0 to 3). We suppose that  $A = A(y)$  and  $\phi = \phi(y)$ .

One can determine the static equations of motion for the above system are of the form

$$(\mathcal{L}_X + 2X\mathcal{L}_{XX})\phi'' - (2X\mathcal{L}_{X\phi} - \mathcal{L}_\phi) = -4\mathcal{L}_X A' \phi', \quad (3)$$

$$A'' + 2A'^2 = \frac{2}{3}\mathcal{L}, \quad (4)$$

$$A'^2 = \frac{1}{3}(\mathcal{L} - 2X\mathcal{L}_X), \quad (5)$$

where prime stands for derivate with respect to  $y$ ,  $\mathcal{L}_X = \partial\mathcal{L}/\partial X$  and  $\mathcal{L}_\phi = \partial\mathcal{L}/\partial\phi$ .

Furthermore, the matter energy density is given by

$$\rho(y) = -e^{2A(y)}\mathcal{L}, \quad (6)$$

and the scalar curvature (or Ricci scalar) is given by

$$R = -4(5A'^2 + 2A''). \quad (7)$$

The Lagrangian density  $\mathcal{L}(\phi, X)$  has the form

$$\mathcal{L} = K(X) - V(\phi), \quad (8)$$

where  $K(X)$  and  $V(\phi)$  are nonstandard kinetic term and potential respectively.

For this case, from Eqs. (3), (4) and (5) the equations of motion can be expressed as

$$(K' + 2XK'')\phi'' - V_\phi = -4K'A'\phi', \quad (9)$$

$$A'' + 2A'^2 = \frac{2}{3}(K - V), \quad (10)$$

$$A'^2 = \frac{1}{3}(K - V - 2XK'). \quad (11)$$

These equations are the static equations of motion of a system with nonstandard dynamics.

In [33], the authors present two explicit models for  $K(X)$ , here we review one of these models.

#### A. The model: $K(X) = X + \alpha|X|X$

This model is also considered in [40], where  $\alpha$  is a real non-negative parameter and  $X = -\frac{1}{2}\phi'^2$ . If  $\alpha = 0$  the standard scenario is restored. For this model, the equations of motion are

$$\phi'' + 4A'\phi' - V_\phi = -\alpha(3\phi'' + 4\phi'A')\phi'^2, \quad (12)$$

$$A'' + 2A'^2 = -\frac{1}{3}\left(1 + \frac{\alpha}{2}\phi'^2\right)\phi'^2 - \frac{2}{3}V, \quad (13)$$

$$A'^2 = \frac{1}{6} \left( 1 + \frac{3}{2} \alpha \phi'^2 \right) \phi'^2 - \frac{1}{3} V. \quad (14)$$

It is possible to rewrite (13) and (14) as

$$A'' = -\frac{2}{3} \phi'^2 (1 + \alpha \phi'^2), \quad (15)$$

Now, to extend the first-order framework to the braneworld scenario, we follow the work in Ref. [4], and choose the derivative of the warp factor with respect to the extra dimension to be a function of the scalar field

$$A' = -\frac{1}{3} W(\phi). \quad (16)$$

Substituting (16) into (15), we get

$$\phi' + \alpha \phi'^3 = \frac{1}{2} W_\phi, \quad (17)$$

this equation is a cubic equation in  $\phi'$ , then the real solution to this cubic equation is given by

$$\phi' = \frac{m(W_\phi)}{6\alpha} - \frac{2}{m(W_\phi)}, \quad (18)$$

where

$$m(W_\phi) = \left( 54\alpha^2 W_\phi + 6\sqrt{3} (16\alpha^3 + 27\alpha^4 W_\phi^2)^{1/2} \right)^{1/3}. \quad (19)$$

It is instructive to note that the equation (18) is the first-order differential equation for the scalar field  $\phi$ .

The potential is obtained by substituting (16) in (14)

$$V(\phi) = \frac{1}{2} \phi'^2 + \frac{3}{4} \alpha \phi'^4 - \frac{1}{3} W(\phi)^2, \quad (20)$$

where  $\phi'$  is given by (18).

On the other hand, we consider the energy functional [4]

$$E[A, \phi] = \int dy (-\mathcal{L}_{system}), \quad (21)$$

where  $\mathcal{L}_{system} = \sqrt{|g|} [-R/4 + \mathcal{L}(\phi, X)]$ . This energy functional is associated with the energy of system, as was reported by Hawking [41]. In our case, (21) becomes

$$E[A, \phi] = \int_{-\infty}^{\infty} dy e^{4A} \left\{ \frac{1}{2} \phi'^2 - 3A'^2 + \frac{\alpha}{4} \phi'^4 + V \right\}. \quad (22)$$

The Euler-Lagrange differential equations from the functional (22) are the static equations of motion (12) and (13), this result is valid for all  $\alpha$ . Otherwise, substituting the potential (20) in (22), for  $\alpha$  which is not necessarily small, we can not deduce the first order differential equations (16) and (18) from the energy functional, therefore the solutions of the first order differential equations not necessarily minimize the energy functional.

At this point, it is also instructive to analyze the matter energy in the model with nonstandard dynamics. From (6), (8) and (20), we get

$$E_\phi = \int_{-\infty}^{\infty} dr e^{2A} \left\{ \phi'^2 + \alpha \phi'^4 - \frac{1}{3} W^2 \right\}. \quad (23)$$

Finally, using (15) and (16) and integrating, we obtain

$$E_\phi = \frac{1}{2} \left( e^{2A(\infty)} W(\phi(\infty)) - e^{2A(-\infty)} W(\phi(-\infty)) \right), \quad (24)$$

the value of the matter energy for all  $\alpha$ . Note that the matter energy depends on the asymptotic behavior of the warp factor.

Now, we follow the same procedure of Ref. [33] and let us focus our study in the case of  $\alpha$  very small. Thus, the solution of (17), up to first-order in  $\alpha$  becomes

$$\phi' = \frac{1}{2} W_\phi - \frac{\alpha}{8} W_\phi^3, \quad (25)$$

and substituting (25) into (20) we obtain the potential

$$V(\phi) = \frac{1}{8} W_\phi^2 - \frac{\alpha}{64} W_\phi^4 - \frac{1}{3} W^2. \quad (26)$$

At this point we turn to examine the energy functional (22). Substituting the potential (26) in (22), we get

$$\begin{aligned} E[A, \phi] = \int_{-\infty}^{\infty} dy \left\{ \frac{1}{2} \left( \phi' - \frac{1}{2} W_\phi + \frac{\alpha}{8} W_\phi^3 \right)^2 - 3 \left( A' + \frac{1}{3} W \right)^2 \right\} \\ + \frac{\alpha}{8} \int_{-\infty}^{\infty} dy \left( 2\phi'^4 + \frac{3}{8} W_\phi^4 - W_\phi^3 \phi' \right) + \frac{1}{2} \int_{-\infty}^{\infty} dy \frac{d}{dy} (W e^{4A}). \end{aligned} \quad (27)$$

From (27) we can conclude that the solutions of the first order differential equations, (16) and (25), are those that minimize the energy functional. Thus, the first order differential equations can be seen as the BPS equations and the energy functional could play the role of a BPS energy in such scenario. The value of the energy system for  $\alpha$  very small is given by

$$E[A, \phi] = \left| e^{4A(\infty)} W(\phi(\infty)) - e^{4A(-\infty)} W(\phi(-\infty)) \right|. \quad (28)$$

Again, the asymptotic behavior of the warp factor plays a leading role in the value of the energy functional.

Now, we find explicitly the solutions for (16) and (25), which minimizes the energy functional. Then, the solution for (25) becomes

$$\phi(y) = \phi_0(y) - \frac{\alpha}{4} W_\phi(\phi_0(y)) W(\phi_0(y)), \quad (29)$$

where  $\phi_0(y)$  is the solution when  $\alpha = 0$ . From (16) and (29), we obtain

$$A(y) = A_0(y) + \frac{\alpha}{12} W(\phi_0(y))^2, \quad (30)$$

where  $A_0(y)$  represents  $A(y)$  when  $\alpha = 0$ . The matter energy density given by (6) is

$$\rho = e^{2A(y)} \left( \frac{1}{4} W_\phi^2 - \frac{1}{3} W^2 - \frac{\alpha}{16} W_\phi^4 \right), \quad (31)$$

substituting (29) and (30) in (31), we obtain

$$\rho = \rho_0 - \frac{\alpha}{48} e^{2A_0(y)} \left( 6W_{\phi\phi} W_\phi^2 W - 10W^2 W_\phi^2 + 3W_\phi^4 + \frac{8}{3} W^4 \right)_{\phi=\phi_0}, \quad (32)$$

where

$$\rho_0 = e^{2A_0(y)} \left( \frac{1}{4} W_\phi^2 - \frac{1}{3} W^2 \right)_{\phi=\phi_0}, \quad (33)$$

note that the energy density (32) is a little bit different from that given in Ref. [33].

To show the validity of the solutions, (29), (30) and (32), we consider the superpotencial  $W(\phi)$  of the form [6]

$$W(\phi) = 3bc \sin \left( \sqrt{\frac{2}{3b}} \phi \right), \quad (34)$$

where  $b$  and  $c$  are positive parameters. The classical solutions for (29) and (30) are given by

$$\phi(y) = \sqrt{\frac{3b}{2}} \arcsin [\tanh(cy)] - \frac{3\sqrt{6}\alpha}{4} b^{3/2} c^2 \tanh(cy) \operatorname{sech}(cy), \quad (35)$$

and

$$A(y) = b \ln [\operatorname{sech}(cy)] + \frac{3\alpha}{4} b^2 c^2 \tanh^2(cy). \quad (36)$$

The profiles of the matter energy density is shown in Fig. (1) for some values of  $\alpha$ . As in Ref. [33] the numerical study gives full support to the analytical expressions for  $\alpha$  very small. The Fig. (1) clearly shows that the brane is localized at  $y = 0$ , because this region has a positive matter energy density. The contribution of the nonstandard kinetic term modifies

the profile without altering the symmetrical form of matter energy density, as noted in Fig. (1). It is known that the  $\alpha$ -parameter, which indicates the strength of fourth-order kinetic term, is used to modify the dynamics of the scalar field contributes to thicker the brane, as reported in [33]. The profiles of the matter energy density and the Ricci scalar are shown in Fig. (2) for  $\alpha = 0.1$ . Note that the presence of regions with positive Ricci scalar is connected with the localization of the brane and it reinforces the conclusion of the analyzes from the matter energy density. Also note that far from the brane,  $R$  tends to a negative constant, characterizing the  $AdS_5$  limit from the bulk, as reported in [27]. A similar behavior is obtained for  $\alpha = 1$  and  $\alpha = 10$ . The profiles of the warp factor is shown in Fig. (3) for some values of  $\alpha$ . Fig. (3) shows that  $e^{2A} \rightarrow 0$  as  $y \rightarrow \pm\infty$ , therefore the matter energy (24) and the energy of system (28) both are zero.

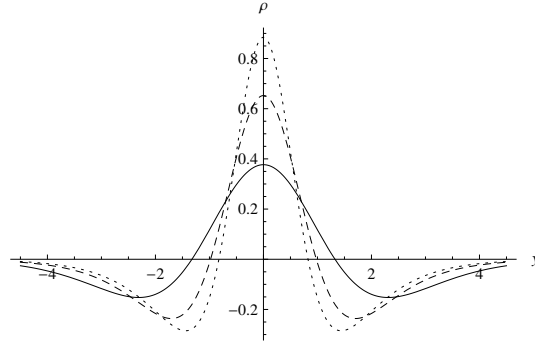


FIG. 1: The profiles of the energy density for  $b = 2/3$ ,  $c = 1$ ,  $\alpha = 0.1$  (dotted line),  $\alpha = 1$  (dashed line) and  $\alpha = 10$  (thin line).

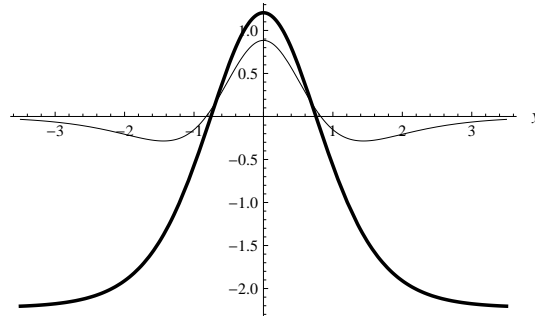


FIG. 2: The profiles of the matter energy density (thin line) and Ricci scalar (thick line) for  $b = 2/3$ ,  $c = 1$  and  $\alpha = 0.1$ .



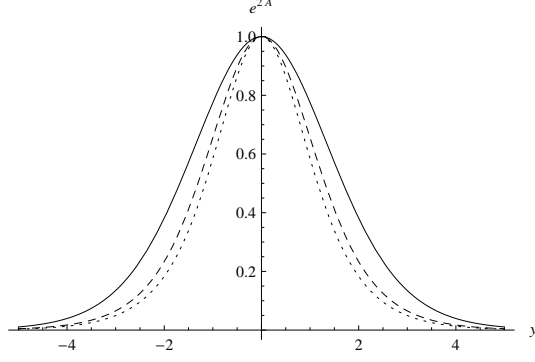


FIG. 3: The profiles of the warp factor for  $b = 2/3$ ,  $c = 1$ ,  $\alpha = 0.1$  (dotted line),  $\alpha = 1$  (dashed line) and  $\alpha = 10$  (thin line).

Now on, we can analyze the exact solutions in the thin brane limit ( $c \rightarrow \infty$ ), we obtain

$$\phi(y) = \frac{\sqrt{6b}}{4} \pi \operatorname{sgn}(y), \quad (37)$$

$$A(y) = -bc|y|, \quad (38)$$

from this we can conclude that the braneworld model with nonstandard dynamics treated here is consistent, because it reduces to the Randall-Sundrum model (thin brane model).

### III. FERMION LOCALIZATION

The action for a Dirac spinor field coupled with the scalar fields by a general Yukawa coupling is

$$S = \int d^5x \sqrt{|g|} [i\bar{\Psi}\Gamma^M\nabla_M\Psi - \eta\bar{\Psi}F(\phi)\Psi], \quad (39)$$

where  $\eta$  is the positive coupling constant between fermions and the scalar field. Moreover, we are considering the covariant derivative  $\nabla_M = \partial_M + \frac{1}{4}\omega_M^{\bar{A}\bar{B}}\Gamma_{\bar{A}}\Gamma_{\bar{B}}$ , where  $\bar{A}$  and  $\bar{B}$ , denote the local Lorentz indices and  $\omega_M^{\bar{A}\bar{B}}$  is the spin connection. Here we consider the field  $\phi$  as a background field. The equation of motion is obtained as

$$i\Gamma^M\nabla_M\Psi - \eta F(\phi)\Psi = 0. \quad (40)$$

At this stage, it is useful to consider the fermionic current. The conservation law for  $J^M$  follows from the standard procedure and it becomes

$$\nabla_M J^M = \bar{\Psi}(\nabla_M\Gamma^M)\Psi, \quad (41)$$

where  $J^M = \bar{\Psi}\Gamma^M\Psi$ . Thus, if

$$\nabla_M\Gamma^M = 0, \quad (42)$$

then four-current will be conserved. The condition (42) is the purely geometrical assertion that the curved-space gamma matrices are covariantly constant.

Using the same line element (2) and the representation for gamma matrices  $\Gamma^M = (e^{-A}\gamma^\mu, -i\gamma^5)$ , the condition (42) is trivially satisfied and therefore the current is conserved.

The equation of motion (40) becomes

$$[i\gamma^\mu\partial_\mu + \gamma^5 e^A(\partial_y + 2\partial_y A) - \eta e^A F(\phi)] \Psi = 0. \quad (43)$$

Now, we use the general chiral decomposition

$$\Psi(x, y) = \sum_n \psi_{L_n}(x) \alpha_{L_n}(y) + \sum_n \psi_{R_n}(x) \alpha_{R_n}(y), \quad (44)$$

with  $\psi_{L_n}(x) = -\gamma^5 \psi_{L_n}(x)$  and  $\psi_{R_n}(x) = \gamma^5 \psi_{R_n}(x)$ . With this decomposition  $\psi_{L_n}(x)$  and  $\psi_{R_n}(x)$  are the left-handed and right-handed components of the four-dimensional spinor field, respectively. After applying (44) in (43), and demanding that  $i\gamma^\mu\partial_\mu\psi_{L_n} = m_n\psi_{R_n}$  and  $i\gamma^\mu\partial_\mu\psi_{R_n} = m_n\psi_{L_n}$ , we obtain two equations for  $\alpha_{L_n}$  and  $\alpha_{R_n}$

$$[\partial_y + 2\partial_y A + \eta F(\phi)] \alpha_{L_n} = m_n e^{-A} \alpha_{R_n}, \quad (45)$$

$$[\partial_y + 2\partial_y A - \eta F(\phi)] \alpha_{R_n} = -m_n e^{-A} \alpha_{L_n}. \quad (46)$$

Inserting the general chiral decomposition (44) into the action (39), using (45) and (46) and also requiring that the result take the form of the standard four-dimensional action for the massive chiral fermions

$$S = \sum_n \int d^4x \bar{\psi}_n (\gamma^\mu \partial_\mu - m_n) \psi_n, \quad (47)$$

where  $\psi_n = \psi_{L_n} + \psi_{R_n}$  and  $m_n \geq 0$ , the functions  $\alpha_{L_n}$  and  $\alpha_{R_n}$  must obey the following orthonormality conditions

$$\int_{-\infty}^{\infty} dy e^{3A} \alpha_{L_m} \alpha_{R_n} = \delta_{LR} \delta_{mn}. \quad (48)$$

Implementing the change of variables

$$z = \int_0^y e^{-A(y')} dy', \quad (49)$$

$\alpha_{L_n} = e^{-2A} L_n$  and  $\alpha_{R_n} = e^{-2A} R_n$ , we get

$$-L_n''(z) + V_L(z) L_n = m_n^2 L_n, \quad (50)$$

$$-R_n''(z) + V_L(z)R_n = m_n^2 R_n, \quad (51)$$

where

$$V_L(z) = \eta^2 e^{2A} F^2(\phi) - \eta \partial_z (e^A F(\phi)), \quad (52)$$

$$V_R(z) = \eta^2 e^{2A} F^2(\phi) + \eta \partial_z (e^A F(\phi)). \quad (53)$$

Using the expressions  $\partial_z A = e^{A(y)} \partial_y A$  and  $\partial_z F = e^{A(y)} \partial_y F$ , we can recast the potentials (52) and (53) as a function of  $y$  [28],[29]

$$V_L(z(y)) = \eta e^{2A} [\eta F^2 - \partial_y F - F \partial_y A(y)], \quad (54)$$

$$V_R(z(y)) = V_L(z(y))|_{\eta \rightarrow -\eta}. \quad (55)$$

It is worthwhile to note that we can construct the Schrödinger potentials  $V_L$  and  $V_R$  from eqs. (54) and (55).

At this stage, it is instructive to state that with the change of variable (49) we get a geometry to be conformally flat

$$ds^2 = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2). \quad (56)$$

Now we focus attention on the condition (42) for the line element (56). In this case we obtain

$$\nabla_M \Gamma^M = i(\partial_z A(z)) e^{-A(z)} \gamma^5. \quad (57)$$

Therefore, the current is no longer conserved for the line element (56). It is known that, in general, the reformulation of the theory in a new conformal frame leads to a different, physically inequivalent theory. This issue has already a precedent in cosmological models [42]. Recently, other inconsistencies are stated in [30].

Under this arguments, we only use the change of variable (49) to have a qualitative analysis of the potential profiles (54) and (55), which is a fundamental ingredient for the fermion localization on the brane.

Now we focus attention on the calculation of the zero mode. Substituting  $m_n = 0$  in (45) and (46) and using  $\alpha_{L_n} = e^{-2A} L_n$  and  $\alpha_{R_n} = e^{-2A} R_n$ , respectively, we get

$$L_0 \propto \exp \left[ -\eta \int_0^y dy' F(\phi) \right], \quad (58)$$

$$R_0 \propto \exp \left[ \eta \int_0^y dy' F(\phi) \right]. \quad (59)$$

This fact is the same to the case of two-dimensional Dirac equation [43]. At this point is worthwhile to mention that the normalization of the zero mode and the existence of a minimum for the effective potential at the localization on the brane are essential conditions for the problem of fermion localization on the brane. This fact was already reported in [29].

In order to guarantee the normalization condition (48) for the left-handed fermion zero mode (58), the integral must be convergent, *i.e*

$$\int_{-\infty}^{\infty} dy \exp \left[ -A(y) - 2\eta \int_0^y dy' F(\phi(y')) \right] < \infty. \quad (60)$$

This result clearly shows that the normalization of the zero mode is decided by the asymptotic behavior of  $F(\phi(y))$ . Furthermore, from (54) and (55), it can be observed that the effective potential profile depends on the  $F(\phi(y))$  choice. This fact implies that the existence of a minimum for the effective potential  $V_L(z(y))$  or  $V_R(z(y))$  at the localization on the brane is decided by  $F(\phi(y))$ . This point will be more clear when it is considered a specific Yukawa coupling. Therefore, the behavior of  $F(\phi(y))$  plays a leading role for the fermion localization on the brane [29]. Having set up the two essential conditions for the problem of fermion localization on the brane, we are now in a position to choose some specific forms for Yukawa couplings.

### A. Zero mode and fermion localization

From now on, we mainly consider the simplest case  $F(\phi) = \phi$ . First, we consider the normalizable problem of the solution. In this case, from Eqs. (35) and (36) the integrand in (60) can be expressed as

$$I = \exp \left[ -b \ln (\operatorname{sech}(cy)) - \frac{3\alpha}{4} b^2 c^2 \tanh^2(cy) - \eta \sqrt{6b} \bar{I}(y) - \eta \frac{3\sqrt{6}\alpha}{2} b^{3/2} c \operatorname{sech}(cy) \right], \quad (61)$$

where  $\bar{I} = \int dy' \arcsin [\tanh(cy')]$ . Following the same procedure of Ref. [18], we only need to consider the asymptotic behavior of the integrand. It becomes

$$I \rightarrow \exp \left[ - \left( \eta \pi \sqrt{\frac{3b}{2}} - bc \right) |y| \right]. \quad (62)$$

This result clearly shows that the zero mode of the left-handed fermions is normalized only for  $\eta > \frac{c}{\pi} \sqrt{\frac{2b}{3}}$ . Note that the asymptotic behavior of the normalization condition for this

case is independent of  $\alpha$ . Now, under the change  $\eta \rightarrow -\eta$  ( $L_0 \rightarrow R_0$ ) we obtain that the right-handed fermions can not be a normalizable zero mode. The shape of the potentials for this case are shown in Fig. (4) for some values of  $\alpha$ . The Fig. (4a) shows that the potential of left-handed fermions,  $V_L$ , is indeed a volcano-like potential. The shapes of the matter energy density,  $V_L$  potential and zero mode for this case are shown in Fig. (5) for  $\alpha = 0.1$ . A similar behavior is obtained for  $\alpha = 1$  and  $\alpha = 10$ . The Fig. (5) clearly shows that the effective potential  $V_L$  has a minimum at the localization of the brane, therefore, this clearly shows that the zero mode of the left-handed fermions is localized on the brane. On the other hand, the figure (4a) shows that the depth of the well structure decreases as  $\alpha$  increases. From this can be conclude that the ability to trap fermions of the effective potential  $V_L$  is inversely proportional to  $\alpha$ . Figure (4b) shows that the potential  $V_R$  is always positive. This effective potential has a maximum that decreases as  $\alpha$  increases. Therefore, none bound fermions with right chirality can not be trapped by the potential  $V_R$ . The analytic expressions for this model are only valid for  $\alpha$  small. For  $\alpha$ , which is not necessarily small, the numerical study is used. The numerical study done for a large range of values of  $\alpha$  bear out our results.

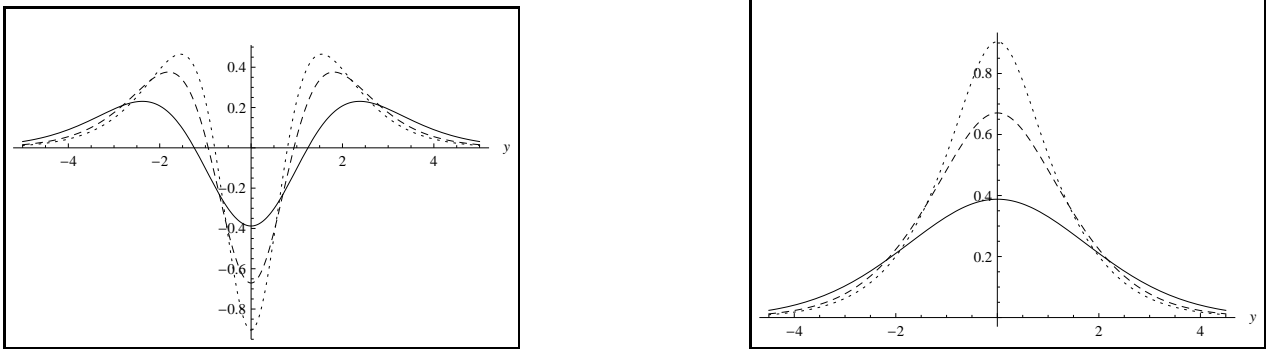


FIG. 4: Potential profile: (a)  $(V_L(y))_A$  (left) and (b)  $(V_R(y))_A$  (right) for  $\eta = 1$ ,  $b = 2/3$ ,  $c = 1$ ,  $\alpha = 0.1$  (dotted line),  $\alpha = 1$  (dashed line) and  $\alpha = 10$  (thin line)

#### IV. CONCLUSIONS

We have reinvestigated the braneworld model constructed out from one scalar field with nonstandard kinetic terms coupled with gravity. We have considered the model  $\mathcal{L} = K(X) - V(\phi)$ , where  $K = X + \alpha|X|X$  (type I model in Ref. [33]). This work completes the analysis

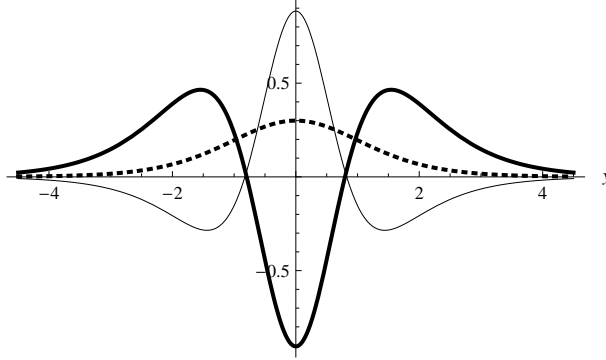


FIG. 5: The profiles of the energy density (thin line),  $(V_L)_A$  (thick line) and zero mode (dashed thick line) for  $b = 2/3$ ,  $c = 1$ ,  $\eta = 1$  and  $\alpha = 0.1$ .

of the research in Ref. [33]. We showed that the equations of motion for all  $\alpha$  can be deduced from the functional  $E[A, \phi]$  (22), as done by Townsend [4] in the case of standard dynamics. Furthermore, we showed that for  $\alpha$  small the solutions of the first order differential equations, (16) and (25), are those that minimize the energy of system. In contrast, for  $\alpha$ , which is not necessarily small, the solutions of the first order differential equations not necessarily minimize the energy of system. Also, we showed that the value of the matter energy and the energy of system depends on the asymptotic behavior of the warp factor. We found an expression for the matter energy density that differs slightly from [33]. The numerical study gives full support to our matter energy density expression for  $\alpha$  small.

We also have investigated the localization problem of fermions for the type I model. We have used the simplest Yukawa coupling  $\eta \bar{\Psi} \phi \Psi$  between the scalar and the spinor fields. In order to guarantee the normalization condition for the zero mode, we showed that the zero mode of left-handed fermions is normalizable under the condition  $\eta > \frac{c}{\pi} \sqrt{\frac{2b}{3}}$  and it is independent of  $\alpha$ . For this kind of solution, the effective potential of left-handed fermions  $V_L$  is a volcano-like potential.  $V_L$  has a minimum at the localization of the brane ( $y = 0$ ), therefore the zero mode of the left-handed is localized on the brane. On the other hand, the value of  $\alpha$  adjust the minimum of  $V_L$ , the depth of the well structure decreases as  $\alpha$  increases. Therefore, we can conclude that the ability to trap fermions of  $V_L$  is inversely proportional to  $\alpha$ . The right-handed fermions can not be localized on the brane, this fact is a consequence of the absence of a normalizable zero mode. For  $\alpha$  not necessarily small, the numerical study done for a large range of values of  $\alpha$  bear out our results and conclusions.

Additionally, we showed that the change of variable  $dz = e^{-A(y)}dy$  leads to a non conserved current, because the curved-space gamma matrices are not covariantly constant. The effects of non-conserved current in the issue of massive modes is currently under consideration and will be the subject of another thorough study.

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